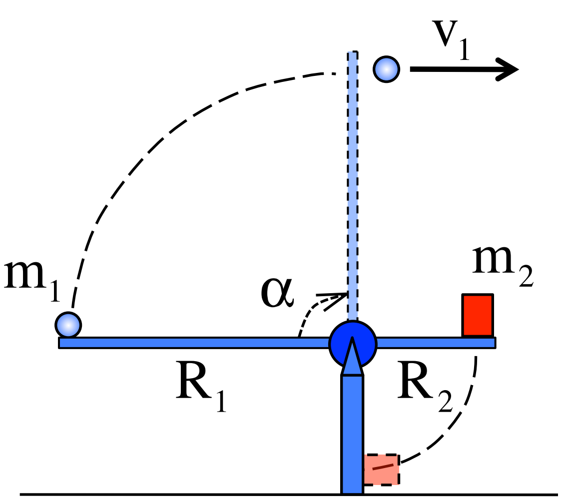
Problem 10.27

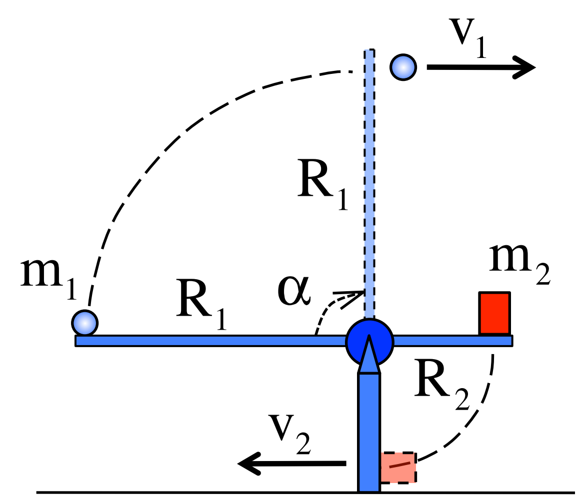
A trebuchet is a medieval sling shot that was used to batter the walls of castles under siege. The idea was that the mass on the left be accelerated by the unequal weight on the right until, at the top of the arc, it comes free to fly.



a.) What is "m_1's" speed at the top (where it is maximum)?

My first inclination was to sum the torques acting on the system, using N.S.L. to determine the angular acceleration, then use kinematics to determine the angular speed and, finally, use the relationship between speed and angular speed to determine the speed at the top. Fortunately for you, you haven't yet run into torque calculations so we'll have to use the much easier and straightforward approach of *conservation of energy*. This process will exactly mirror your use of that approach for a purely translational situation with the exception that there will be rotational kinetic energy in the mix. Keeping that in mind:

Let's assume the *zero gravitational potential energy level* is at the pin (i.e., where everything is sitting at the start). Remembering that the relationship between a body's *angular speed* and its *translational speed* is $v = R\omega$, and treating each mass like a point mass, the *conservation of energy* allows us to write:



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + 0 + 0 = \left(\frac{1}{2}m_1 v_1^{\ 2} + \frac{1}{2}m_1 v_2^{\ 2}\right) + \left(m_1 g R_1 + m_2 g(-R_2)\right)$$

To relate the speeds, we need to note that although the *speeds* of each is different, the *angular speeds* of each *are the same*. As such, we can write:

$$\omega_1 = \frac{v_1}{R_1} = \frac{v_2}{R_2}$$

$$\Rightarrow \quad v_2 = \left(\frac{v_1}{R_1}\right)R_2$$

Re-writing the *conservation of energy* relationship with that, we get:

$$0=\left(\frac{1}{2}m_1v_1^{\ 2}+\frac{1}{2}m_2v_2^{\ 2}\right)+\left(m_1gR_1+m_2g\left(-R_2\right)\right)$$

$$\Rightarrow \quad \left(\frac{1}{2}m_1v_1^{\ 2}+\frac{1}{2}m_2\left(\frac{R_2}{R_1}v_1\right)^2\right)=-\left(m_1gR_1+m_2g\left(-R_2\right)\right)$$

$$\Rightarrow \quad v_1=\left(\frac{2\left(-m_1gR_1+m_2gR_2\right)}{\left(m_1+m_2\left(\frac{R_2}{R_1}\right)^2\right)}\right)^{1/2}$$

$$\Rightarrow \quad v_1=\left(\frac{2\left(-(.120\text{ kg})(9.8\text{ m/s})(3.00\text{ m}-.140\text{ m})+(60.0\text{ kg})(9.8\text{ m/s})(.140\text{ m})\right)}{\left((.120\text{ kg})+(60.0\text{ kg})\left(\frac{(.140\text{ m})}{(3.00-.12\text{ m})}\right)^2\right)}\right)^{1/2}$$

$$\Rightarrow \quad v_1=24.5\text{ m/s}$$

Note: If you had decided to treat this problem as though the bodies were moving through a pure rotation about the pivot, the the kinetic energy would be rotational ($\left(\frac{1}{2}\right)I\omega^2$) with *I* for a point mass being mr^2. With that, you could write the *conservation of energy* relationship as:

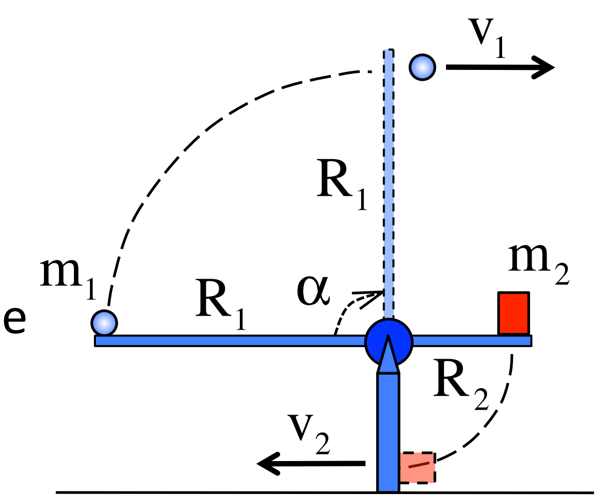
$$0=\left(\frac{1}{2}I_1\omega_1^{\ 2}+\frac{1}{2}I_2\omega_1^{\ 2}\right)+\left(m_1gR_1+m_2g\left(-R_2\right)\right)$$

$$\Rightarrow\quad \left(\frac{1}{2}\left(m_1R_1^{\ 2}\right)\omega_1^{\ 2}+\frac{1}{2}\left(m_2R_2^{\ 2}\right)\omega_1^{\ 2}\right)=-\left(m_1gR_1+m_2g\left(-R_2\right)\right)$$

$$\Rightarrow\quad \omega_1=\left(\frac{2\left(-m_1gR_1+m_2gR_2\right)}{\left(m_1R_1^{\ 2}+m_2R_2^{\ 2}\right)}\right)^{1/2}$$

Once you had the angular speed, you could use $v=R\omega$ to determine the exit speed for the small mass.

b.) Is the total *acceleration* constant?



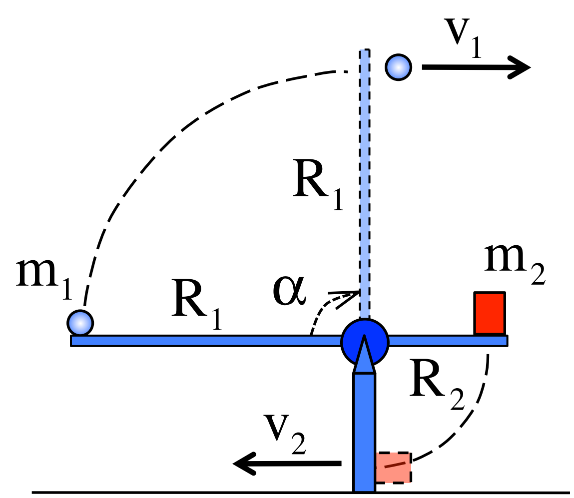
The text's Solution Manual points out that for the bodies to be moving with constant acceleration, they would either have to be moving in straight-line motion or following a parabolic arc (presumably because a parabolic *position versus time* graph yields an acceleration that doesn't change with time —think gravity. This is obscure but true). A better answer, should this come up on an AP test where you have had full exposure to all the material you will encounter in the rotational motion chapter, is to note that the torque on the system is due to gravity and is not constant (in the beginning position, the torque due to gravity about the pin will be large; at the top, it will be zero). This will make more sense to you after you've studied torque and the rotational version of N.S.L., but the bottom line is that if the *torque* isn't constant, the *angular acceleration* won't be constant. And if that's true, the *translational acceleration* won't be constant, either.

c.) Do the bodies move with constant *tangential acceleration*?

Again, if the *angular acceleration* isn't constant, the *translational acceleration* won't be constant, either.

d.) Is the *angular acceleration* constant?

No. Answer justified in *Part b*.



e.) Is the trebuchet's *momentum* constant?

No. The rotational version of momentum, called (cleverly enough) angular momentum, is:

$$\sum I_1\omega_1 + \sum \Gamma_{ext}\Delta t = \sum I_2\omega_2$$

This rather obscure bit of amusement states that if the total initial angular momentum ($I\omega$) isn't altered by an external rotational impulse (the torque times *time* quantity), the final angular momentum will be the same as the initial. In this case, gravity is producing an external torque on the system, so angular momentum is NOT conserved.

f.) Does the trebuchet/earth system have constant *mechanical energy*?

Yes, the system is conserved in mechanical energy. With the earth in the system, gravity is an internal force whose work quantity is dealt with using the gravitational potential energy (that's how we solved *Part a*).